



Optimal Production Inventory Model with Preservation Technology, Recycling, and Partial Backlogging under Level Dependent Demand

Mohammad Ekramol Islam¹, Md. Asrafi Al Mamun^{2*}

¹ Professor of Mathematics & Treasure, Sonargaon University, Dhaka, Bangladesh

^{2*} Department of Mathematics, Birshreshtha Munshi Abdur Rouf Public College, Dhaka, Bangladesh & M.Phil Research Scholar, Department of Mathematics, Jagannath University, Dhaka

*Email: aam.kanon77@gmail.com

Received: April 27, 2026

Revised: May 30, 2026

Accepted: May 29, 2026

Keywords:

Perishable Inventory; Level-Dependent Demand; Cost optimization, Decay Cost; Preservation technology

ABSTRACT

This paper proposes a production inventory model for deteriorating items with level-dependent demand, explicitly incorporating preservation technology. Demand increases with on-hand inventory, reflecting visibility and display effects, while deterioration is reduced through a controllable preservation effort at an additional cost. The model allows shortages, where unmet demand is partially backlogged and partially lost. A comprehensive total cost function is formulated by accounting for production, holding, deterioration, preservation, setup, backlog, and lost-sales costs over a finite production cycle. Owing to the nonlinear and interdependent nature of the system, the optimal production quantity and preservation effort are obtained using numerical optimization techniques. Sensitivity analysis is conducted to examine the impact of key parameters on total cost and decision variables. Numerical results demonstrate that moderate investment in preservation significantly reduces deterioration and overall cost, whereas excessive preservation yields diminishing economic returns. The model provides practical insights for managing production systems dealing with perishable or quality-sensitive products.

1.0 INTRODUCTION

Inventory systems for perishable and deteriorating products pose significant challenges due to quality degradation, demand uncertainty, and the risk of shortages or excess stock. These challenges are particularly pronounced in industries such as food processing, pharmaceuticals, chemicals, and agricultural supply chains, where deterioration directly affects both service level and profitability. Traditional production-inventory models,

which often assume constant demand and negligible deterioration, are inadequate for capturing these practical complexities. In recent years, increasing attention has been given to level-dependent demand, where customer demand is influenced by the available inventory, reflecting display effects and customer perception. At the same time, partial backlogging has emerged as a more realistic representation of shortage behavior, acknowledging that only a fraction of

unmet demand is willing to wait, while the remainder results in lost sales. Additionally, advancements in preservation technology such as improved packaging, temperature control, and chemical treatment offer firms the ability to reduce deterioration rates through targeted investment, though at an additional cost.

Motivated by these considerations, this paper develops a production–inventory model that integrates level-dependent demand, partial backlogging, and preservation effort within a unified framework. The model explicitly captures the interaction between production decisions, deterioration dynamics, and preservation investment. By deriving the total cost function and analyzing its convexity, the study provides a rigorous basis for determining optimal production quantity and preservation effort, along with valuable managerial insights into sustainable and cost-effective inventory management.

2.0 LITERATURE REVIEW

The study of inventory systems for deteriorating and perishable items has a long and well-established history in operations research. The foundational work of (Ghare, 1963) introduced the concept of exponentially decaying inventory, laying the groundwork for modeling deterioration explicitly within inventory dynamics. This seminal contribution motivated extensive subsequent research addressing more realistic demand patterns, production policies, and shortage structures. Early developments focused on integrating deterioration with time-varying demand and production rates. (Goyal and Giri, 2003) systematically analyzed production–inventory systems with simultaneous variations in demand, production, and deterioration, demonstrating the managerial significance of coordinating these factors. Related extensions incorporated partial backlogging and payment delays (Chang and Dye, 2001, 2006), as well as mixtures of backorders and lost sales (Padmanabhan and VRAT, 1990), thereby enhancing practical relevance.

A substantial stream of literature examined demand structures dependent on time, inventory level, or both. (Urban, 2005) provided a comprehensive review and unifying framework for inventory-level-dependent demand, which later studies adapted to deteriorating environments (Balkhi and Benkherouf, 2004; Roy and Chaudhuri, 2006). Models with nonlinear or time-quadratic demand further enriched analytical realism (Jagadeeswari and Chenniappan, 2014; Skouri and Papachristos, 2003).

Extensions to EOQ and EPQ settings with partial backordering and nonlinear holding costs were also investigated [San-José et al., 2015; Min et al., 2012]. With increasing attention to perishability and production systems, researchers developed EPQ-type models incorporating deterioration and production-dependent demand. (Mishra et al., 2013) and (Sharma et al., 2022) analyzed production–inventory systems with time- and stock-dependent demand under partial backlogging, highlighting the operational consequences of demand responsiveness. More recent studies by (Al Mamun and Islam, 2025) and (Islam et al., 2025) focused explicitly on perishable production systems with level-dependent or exponential demand and price incentives during the production run, emphasizing promotional and pricing strategies in production environments.

Another important research direction concerns controllable deterioration and preservation technology. (Hsu et al., 2010) pioneered the integration of preservation

technology investment into deteriorating inventory models, later extended by (Mishra et al., 2017), (Li et al., 2019) and (Zhang et al., 2022) to include stock-dependent demand, nonlinear holding costs, and pricing decisions. Sustainability and green considerations were incorporated through investments in preservation and environmentally responsible operations (Saha et al., 2017; Yadav et al., 2025) reflecting growing concerns about sustainable supply chains.

Recent literature has increasingly addressed complex and realistic operational settings. Studies have incorporated inflation, trade credit, and post-pandemic disruptions (De et al., 2024) multiple trade facilities and full backlogging (Sarkar et al., 2024) trapezoidal or hybrid demand patterns (Kaushik, 2024, Rana and Kumar, 2024) and two-warehouse systems. Comprehensive reviews, such as (Karimi, 2025), confirm the rapid expansion of deteriorating inventory research and highlight the need for integrated models that simultaneously consider demand structure, production policies, and perishability. Overall, existing research demonstrates significant progress in modeling deteriorating and perishable inventory systems under diverse operational assumptions. However, gaps remain in jointly analyzing perishable production systems with exponential or level-dependent demand, constant production rates, and pricing or discount strategies during the production run. The present study contributes to this literature by addressing these gaps and providing additional managerial insights into cost behavior and optimal policy design for perishable production–inventory systems.

3.0 NOTATIONS AND ASSUMPTIONS

3.1 Notations:

The notations we have used to develop this model are shown in table 1

Table-1: Notations and their explanation

Notations	Explanation
K	Set up cost
D	Demand rate where $D = \begin{cases} \gamma + \beta I(t); & t_b \leq t \leq t_s \\ \gamma; & t \in]t_b, t_s[\end{cases}$
γ	Initial demand
δ	Stock dependent coefficient
B_0	Initial backlog
θ	Production rate which exceeds the demand
C_p	Unit Production cost
h	Holding cost
C_b	Backlogging cost
C_l	Lost sales cost
C_u	Preservation cost per unit
C_d	Deterioration cost
μ_0	Deterioration rate without preservation
η	Effectiveness coefficient of preservation technology where $\eta > 0$
ω	Backlogging sensitivity parameter
u	Investment to preservation technology
Q	Maximum inventory level
t_b	Time to clear initial backlog
t_1	Inventory reaches its maximum level
t_s	Time of shortage accumulation
I_1	Inventory level for $[t_b, t_1]$
I_2	Inventory level for $[t_1, t_s]$
Functions	Explanation
$\mu(u)$	Effective deterioration rate
$T(u, Q)$	Cycle time
$TC(u, Q)$	Total cost per unit time
$\mu = \mu(u) = \mu_0 e^{-\eta u}$	Effective deterioration rate

3.2 Assumptions:

- ❖ A continuous-review production–inventory system is considered for a single deteriorating item with a finite production rate.
- ❖ Each cycle begins with a known backlog carried over from the previous cycle.
- ❖ Demand $\gamma + \delta I(t)$ depends on inventory level, where $\gamma = 0, 1, 2, \dots$ and $0 < \delta < 1$
- ❖ Production starts at a finite rate θ to clear backlog at time t_b . Inventory builds up from t_b to t_1 .
- ❖ At t_1 , production stops, and inventory depletes due to both deterioration and demand fulfillment until t_s .
- ❖ Inventory level hits zero at t_s and shortage occurs

3.3 Mathematical Model Formulation

Based on the assumptions and notations introduced in the previous section, we now develop the mathematical model of the proposed inventory system. The inventory cycle begins with a backlog equal to that carried over from the previous cycle. During the cycle, production, demand, and deterioration interact dynamically, leading to variations in both backlog and inventory levels. To capture these dynamics, the system is described through a set of differential equations representing inventory changes in different phases of the cycle.

The proposed inventory model is shown below in figure-1

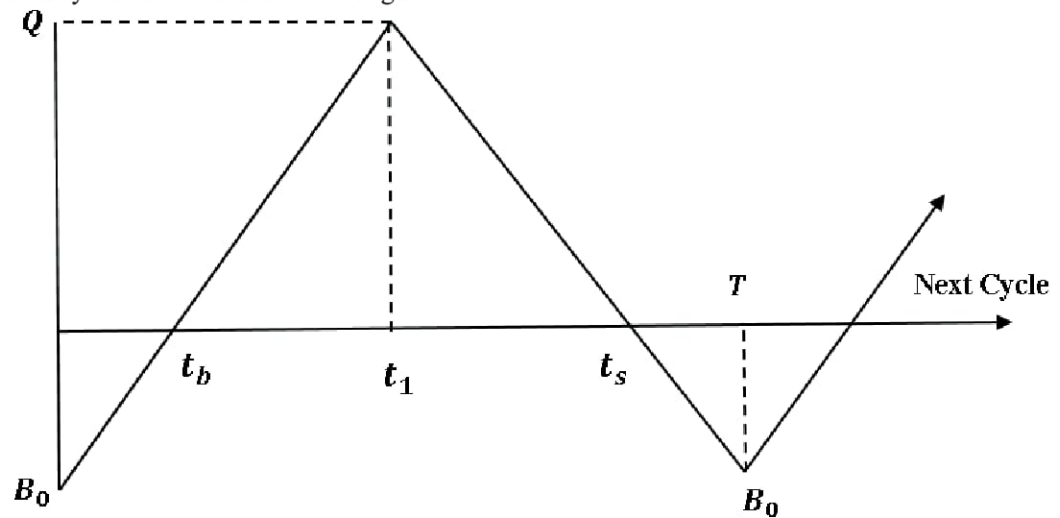


Figure-1: Graphical representation of the proposed model

Phase-1 ($0 \leq t \leq t_b$): Backlog Clearing

Production runs at rate θ where $\theta > \gamma + \delta I(t)$ for clearing backlog. Backlog changes at rate

$$\frac{dB(t)}{dt} = -(\theta - \gamma) \Rightarrow B(t) = (-\theta + \gamma)t + K_1$$

Using boundary condition i.e. at $t = 0$, $B(0) = B_0$ we found $K_1 = B_0$

$$\therefore B(t) = (-\theta + \gamma)t + B_0 \quad (1)$$

Applying another boundary condition ($B(t_b) = 0$) we obtain,

$$t_b = \frac{B_0}{\theta - \gamma} \quad (2)$$

Phase-2 ($t_b \leq t \leq t_1$): Inventory Build up

$$\frac{dI_1(t)}{dt} = \theta - \gamma - (\delta + \mu)I_1(t)$$

$$\Rightarrow I_1(t) = \frac{\theta - \gamma}{\delta + \mu} + K_2 e^{-(\delta + \mu)t}$$

Applying the boundary condition ($I_1(t_b) = 0$),

we obtain the general solution of this differential equation as

$$I_1(t) = \frac{\theta - \gamma}{\delta + \mu} \{1 - e^{-(\delta + \mu)(t - t_b)}\} \quad (3)$$

By another boundary condition that is at $t = t_1$, $I_1(t_1) = Q$ we get

$$\begin{aligned} Q &= \frac{\theta - \gamma}{\delta + \mu} \{1 - e^{-(\delta + \mu)(t_1 - t_b)}\} \\ \Rightarrow t_1 - t_b &= \frac{1}{\delta + \mu} \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} \end{aligned} \quad (4)$$

Phase-3 ($t_1 \leq t \leq t_s$): Inventory Depletion

On the other hand, since no production takes place beyond time t_1 , the inventory decreases at a rate of $\gamma + \delta I_2(t) + \mu I_2(t)$ from $t = t_1$ to t_s . The market's demand and the products' slow degradation cause the inventory to decline. Based in this situation, we can derive the differential equation that follows:

$$\frac{dI_2(t)}{dt} = -\gamma - (\delta + \mu)I_2(t)$$

$$\Rightarrow I_2(t) = -\frac{\gamma}{\delta + \mu} + K_3 e^{-(\delta + \mu)t}$$

By the boundary condition, $I_2(t_1) = Q$, we found the integrating constant

$$\text{as } K_3 = \left(Q + \frac{\gamma}{\delta + \mu}\right) e^{(\delta + \mu)t_1}$$

$$I_2(t) = \left(Q + \frac{\gamma}{\delta + \mu}\right) e^{-(\delta + \mu)(t - t_1)} - \frac{\gamma}{\delta + \mu} \quad (5)$$

Applying another boundary condition which is at $t = t_s$, $I_2(t_s) = 0$, we get

$$\begin{aligned} 0 &= \left(Q + \frac{\gamma}{\delta + \mu}\right) e^{-(\delta + \mu)(t_s - t_1)} - \frac{\gamma}{\delta + \mu} \\ \Rightarrow t_s - t_1 &= \frac{1}{\delta + \mu} \ln \left\{ 1 + \frac{(\delta + \mu)Q}{\gamma} \right\} \end{aligned} \quad (6)$$

Phase-4 ($t_s \leq t \leq T$): Backlog Regeneration:

After inventory hits zero at t_1 , the system enters a backlog accumulation phase and production cannot restart immediately because the system requires a backlog B_0 to initiate the next cycle.

Thus the differential equation for regeneration phase can be expresses as:

$$\frac{dB(t)}{dt} = \gamma \frac{1}{1 + \omega(T - t)}$$

$$\Rightarrow B(t) = -\frac{\gamma}{\omega} \ln \{1 + \omega(T - t_s)\} + K_4$$

Here the boundary condition is $B(t_s) = 0$ provides $K_4 = \frac{\gamma}{\omega} \ln\{1 + \omega(T - t_s)\}$

$$\text{Thus } B(t) = \frac{\gamma}{\omega} \ln\left\{\frac{1+\omega(T-t_s)}{1+\omega(T-t)}\right\} \quad (7)$$

From another condition that is at $t = T$, $B(T) = B_0$ we obtain,

$$\begin{aligned} B_0 &= \frac{\gamma}{\omega} \ln\{1 + \omega(T - t_s)\} \\ \Rightarrow T - t_s &= \frac{e^{\frac{\omega B_0}{\gamma}} - 1}{\omega} \end{aligned} \quad (8)$$

Now using equation (2), (4), (6) and (8) we get the cycle length as

$$\begin{aligned} T &= T - t_s + t_s - t_1 + t_1 - t_b + t_b \\ \Rightarrow T(Q) &= \frac{1}{\omega} \left(e^{\frac{\omega B_0}{\gamma}} - 1 \right) + \frac{1}{\delta + \mu} \ln\left\{1 + \frac{(\delta + \mu)Q}{\gamma}\right\} + \frac{1}{\delta + \mu} \ln\left\{\frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q}\right\} + \frac{B_0}{\theta - \gamma} \end{aligned} \quad (9)$$

And the time to reach inventory to its maximum level

$$\begin{aligned} t_1 &= t_1 - t_b + t_b \\ \Rightarrow t_1(Q) &= \frac{1}{\delta + \mu} \ln\left\{\frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q}\right\} + \frac{B_0}{\theta - \gamma} \end{aligned} \quad (10)$$

Total Cost Function

The total cost per cycle consists of the following cost components:

1. Set up cost = K

2. Production cost over the time interval $[0, t_1]$ is

$$\begin{aligned} PC &= C_p \theta t_1 \\ &= C_p \theta \left[\frac{1}{\delta + \mu} \ln\left\{\frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q}\right\} + \frac{B_0}{\theta - \gamma} \right] \end{aligned} \quad [\text{Using equation (10)}]$$

3. Holding cost during the interval $[t_b, t_s]$ is

$$\begin{aligned} HC &= h \left\{ \int_{t_b}^{t_1} I_1(t) dt + \int_{t_1}^{t_s} I_2(t) dt \right\} \\ &= h \left[\frac{\theta - \gamma}{\delta + \mu} \left\{ (t_1 - t_b) - \frac{1 - e^{-(\delta + \mu)(t_1 - t_b)}}{\delta + \mu} \right\} + \frac{Q}{\delta + \mu} - \frac{\alpha}{\delta + \mu} ((t_s - t_1)) \right] \\ &= \frac{h}{(\delta + \mu)^2} \left[(\theta - \gamma) \ln\left\{\frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q}\right\} - \gamma \ln\left\{1 + \frac{(\delta + \mu)Q}{\gamma}\right\} \right] \end{aligned}$$

4. Deterioration cost for the interval $[t_b, t_s]$ is

$$\begin{aligned} DC &= C_d \mu \left\{ \int_{t_b}^{t_1} I_1(t) dt + \int_{t_1}^{t_s} I_2(t) dt \right\} \\ &= \frac{C_d \mu}{(\delta + \mu)^2} \left[(\theta - \gamma) \ln\left\{\frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q}\right\} - \gamma \ln\left\{1 + \frac{(\delta + \mu)Q}{\gamma}\right\} \right] \end{aligned}$$

5. Backlogging cost for $t \in [0, t_b]$ is

$$\begin{aligned} BC &= C_b \int_0^{t_b} B(t) dt \\ &= \frac{C_b B_0^2}{2(\theta - \gamma)} \end{aligned}$$

6. Lost sales cost in the interval $[t_s, T]$ is

$$\text{Lost sale rate} = \gamma \left\{ 1 - \frac{1}{1 + \omega(T - t)} \right\} = \gamma \left\{ \frac{\omega(T - t)}{1 + \omega(T - t)} \right\}$$

$$\text{Thus lost sale cost } LC = C_l \int_{t_s}^T \gamma \frac{\omega(T - t)}{1 + \omega(T - t)} dt$$

$$= \frac{C_l \gamma}{\omega} \left(e^{\frac{\omega B_0}{\gamma}} - 1 - \frac{\omega B_0}{\gamma} \right)$$

Therefore total cost per time per cycle in terms of u and Q

$$\begin{aligned} TC(u, Q) &= \frac{K + C_u u + PC + HC + DC + BC + LC}{T(Q)} \\ \Rightarrow TC(u, Q) &= \frac{1}{T(Q)} \left[K + C_u u + C_p \theta \left[\frac{1}{\delta + \mu} \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} + \frac{B_0}{\theta - \gamma} \right] + \frac{h + C_d \mu}{(\delta + \mu)^2} \right. \\ &\quad \left[(\theta - \gamma) \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} - \gamma \ln \left\{ 1 + \frac{(\delta + \mu)Q}{\gamma} \right\} \right] \\ &\quad \left. + \frac{C_b B_0^2}{2(\theta - \gamma)} + \frac{C_l \gamma}{\omega} \left(e^{\frac{\omega B_0}{\gamma}} - 1 - \frac{\omega B_0}{\gamma} \right) \right] \end{aligned} \quad (11)$$

The necessary condition for the total average cost function to be minimized is;

$$\begin{aligned} \frac{\partial(TC)}{\partial Q} &= \frac{1}{T^2} \left[\left[\frac{e^{\frac{\omega B_0}{\gamma}} - 1}{\omega} + \frac{1}{\delta + \mu} \ln \left\{ 1 + \frac{(\delta + \mu)Q}{\gamma} \right\} + \frac{1}{\delta + \mu} \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} + \frac{B_0}{\theta - \gamma} \right] \right. \\ &\quad \left[\frac{C_p \theta}{\theta - \gamma - (\delta + \mu)Q} + \frac{h + C_d u}{(\delta + \mu)^2} \left\{ (\theta - \gamma) \frac{(\delta + \mu)}{\theta - \gamma - \lambda Q} - \frac{\gamma(\delta + \mu)}{\gamma + (\delta + \mu)Q} \right\} \right] \\ &\quad - [K + C_u u + C_p \theta \left[\frac{1}{\delta + \mu} \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} + \frac{B_0}{\theta - \gamma} \right] + \frac{h + C_d \mu}{(\delta + \mu)^2} [(\theta - \gamma) \right. \\ &\quad \left. \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} - \gamma \ln \left\{ 1 + \frac{(\delta + \mu)Q}{\gamma} \right\}] + \frac{C_b B_0^2}{2(\theta - \gamma)} \right. \\ &\quad \left. + \frac{C_l \gamma}{\omega} \left(e^{\frac{\omega B_0}{\gamma}} - 1 - \frac{\omega B_0}{\gamma} \right) \right] \left[\frac{1}{\gamma + (\delta + \mu)Q} + \frac{1}{\theta - \gamma - (\delta + \mu)Q} \right] = 0 \end{aligned} \quad (12)$$

and

$$\begin{aligned} \frac{\partial(TC)}{\partial u} &= \frac{1}{T^2} \left[\left[\frac{e^{\frac{\omega B_0}{\gamma}} - 1}{\omega} + \frac{1}{\delta + \mu} \ln \left\{ 1 + \frac{(\delta + \mu)Q}{\gamma} \right\} + \frac{1}{\delta + \mu} \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} + \frac{B_0}{\theta - \gamma} \right] \right. \\ &\quad [C_u - \eta u \left[\frac{C_p \theta}{\lambda^2} \ln \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} + \left\{ \frac{C_d}{(\delta + \mu)^2} - \frac{2(h + C_d u)}{(\delta + \mu)^3} \right\} \right. \\ &\quad \left. \left\{ (\theta - \gamma) \ln \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} - \gamma \ln \left(1 + \frac{(\delta + \mu)Q}{\gamma} \right) \right\} - \frac{h + C_d u}{(\delta + \mu)} \right. \\ &\quad \left. \left. \left\{ \frac{(\theta - \gamma)Q}{\theta - \gamma - (\delta + \mu)Q} - \frac{\gamma Q}{\gamma + (\delta + \mu)Q} \right\} \right] - [K + C_u u + C_p \theta \right. \\ &\quad \left. \left[\frac{1}{\delta + \mu} \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} + \frac{B_0}{\theta - \gamma} \right] + \frac{h + C_d \mu}{(\delta + \mu)^2} \right. \\ &\quad \left. \left. \left[(\theta - \gamma) \ln \left\{ \frac{\theta - \gamma}{\theta - \gamma - (\delta + \mu)Q} \right\} - \gamma \ln \left\{ 1 + \frac{(\delta + \mu)Q}{\gamma} \right\} \right] + \frac{C_b B_0^2}{2(\theta - \gamma)} \right] \right] \end{aligned} \quad (13)$$

$$+ \frac{C_l \gamma}{\omega} \left(e^{\frac{\omega B_0}{\gamma}} - 1 - \frac{\omega B_0}{\gamma} \right) \times \left[-\eta u \left[-\frac{\ln \left\{ \frac{(\theta - \gamma)(\gamma + (\delta + \mu)Q)}{(\delta + \mu)(\theta - \gamma - (\delta + \mu)Q)} \right\}}{(\delta + \mu)^2} \right. \right. \\ \left. \left. + \frac{Q}{\delta + \mu} \left(\frac{1}{\gamma + (\delta + \mu)Q} + \frac{1}{\theta - \gamma - (\delta + \mu)Q} \right) \right] \right] = 0$$

and the equation (12) and (13) satisfies the following sufficient condition

$$\frac{\partial^2(TC)}{\partial u^2} \times \frac{\partial^2(TC)}{\partial Q^2} - \left(\frac{\partial^2(TC)}{\partial u \partial Q} \right)^2 > 0, \frac{\partial^2(TC)}{\partial u^2} > 0 \quad \text{and} \quad \frac{\partial^2(TC)}{\partial Q^2} > 0$$

Since the equation (12) and (13) are transcendental, therefore we solved these equations numerically for a given set of parameters using the Mathematica software and the obtained value of u and Q satisfy the equation (11) to minimize the total average cost. The convexity of the total cost function is shown in the next section.

4. NUMERICAL ILLUSTRATION

In this section, a numerical example is considered to illustrate this model. The following numerical values of parameter in proper unit were considered for numerical, graphical and sensitivity analysis of this model.

$K=800$; $\theta=500$; $\gamma=130$; $\delta=0.05$; $h=2.5$; $C_b=10$; $C_l=10$; $C_p=12$; $\omega=0.08$; $B_0=200$; $C_u=45$;
 $\mu_0=0.25$; $\eta=0.6$; $C_d=20$; $\theta=500$;

The optimum cycle length $T^* = 3.521$, $Q^* = 132.4$, optimum investment to preservation technology $u^* = 3.779$ are the outcomes obtained by using equations (9), (12) and (13) respectively.

For these values of u and Q , the second partial derivative found to be $\frac{\partial^2(TC)}{\partial u^2} = 7.50773 > 0$,

$\frac{\partial^2 TC}{\partial Q^2} = 0.0110651 > 0$ and $\frac{\partial^2(TC)}{\partial u^2} \times \frac{\partial^2(TC)}{\partial Q^2} - \left(\frac{\partial^2(TC)}{\partial u \partial Q} \right)^2 = 0.0487 > 0$. So the values of u^* and Q^* minimize the total average cost and the computed minimum cost obtained from equation (11) is $TC^* = 2080.15$

Details are shown below in Table 2

Q	u	$T(Q)$	$TC(u, Q)$
42.4	2.279	2.613	2120.48
72.4	2.779	2.918	2095.62
102.4	3.279	3.220	2083.59
132.4	3.779	3.521	2080.15
162.4	4.279	3.822	2083.05
192.4	4.779	4.120	2091.02
222.4	5.279	4.418	2103.21

Table 2: Impact of Q , u , T on TC

5.0 GRAPHICAL PROOF OF THE CONVEXITY OF THE TOTAL COST FUNCTION

If we plot the total cost function (11) against some values of $u \in [0.7, 5.7]$ and $Q \in [102.4, 162.4]$ we get strictly convex graph of total cost function TC shown in Figure 4. The convexity of the total cost function TC is shown in Figure 2 with respect to u fixed at $Q = 132.4$. Again the convexity of the total cost function TC is shown in Figure 3 with respect to Q fixed at $u = 3.779$. Both Figure 2 and 3 illustrates that the total cost function is a strictly convex function.

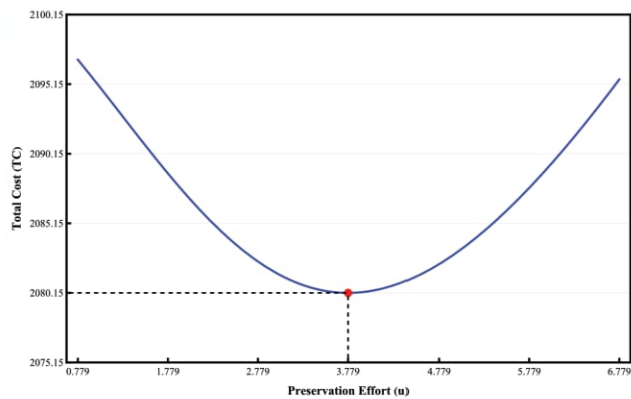


Figure 2: Total cost vs u at $Q^* = 132.4$

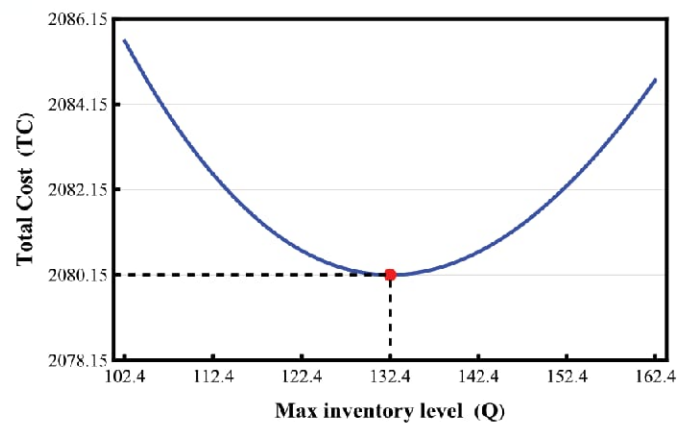


Figure 3: Total cost vs Q at $u^* = 3.779$

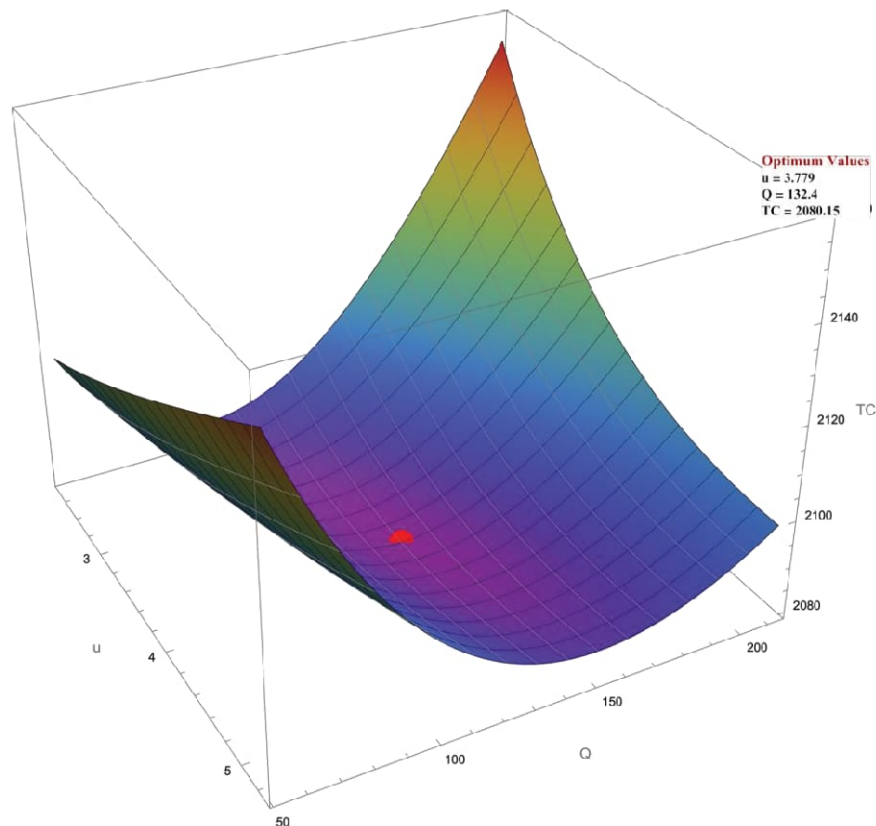


Figure 4: Total cost vs u and Q

The three-dimensional plot of the total cost function $TC(u, Q)$ demonstrates a smooth and well-structured surface with a unique interior minimum over the admissible domain of the decision variables. The cost function increases consistently in all directions as the preservation effort u and the order quantity Q deviate from their optimal values, and no additional local extrema are observed. This behavior is indicative of joint convexity of $TC(u, Q)$ with respect to u and Q . The gradual and continuous curvature of the surface reflects increasing marginal costs associated with both excessive preservation investment and larger production lots, while the cost-reducing effects diminish beyond the optimal point. The existence of a single global minimum confirms that the first-order optimality conditions are sufficient to characterize the optimal policy. Consequently, the model yields a stable and economically meaningful solution, and the optimal decisions are robust to small perturbations in the decision variables.

6.0 SENSITIVITY ANALYSIS

This section will now show how even slight changes to the parameters C_u , $C\mu_0$, η , θ , affect the inventory system or the solution

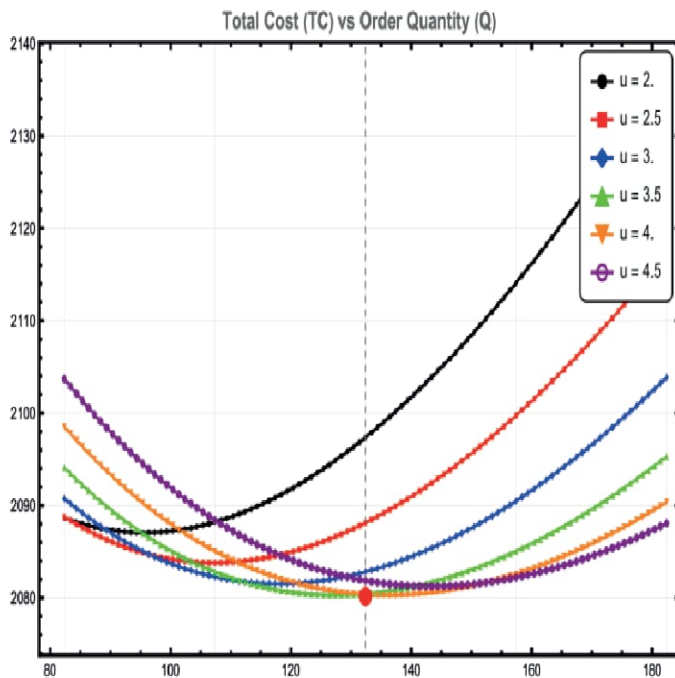


Figure 5(a): TC vs Q for different u values

getting information from households, fisherfolk,

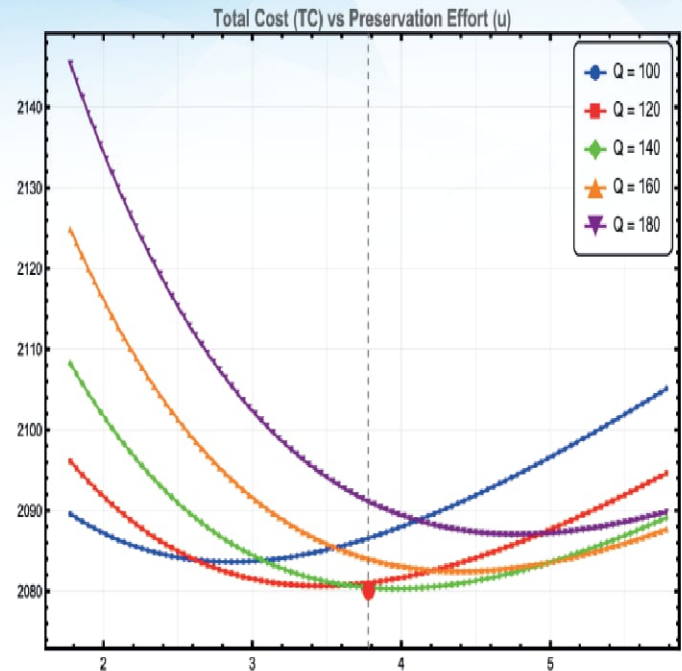


Figure 5(b): TC vs u for different Q values

Figure 5(a) shows that the total cost function is convex in terms of Q for all considered preservation levels u . Each curve attains a unique minimum, confirming the existence of an optimal Q . As u increases, the minimum total cost decreases and the cost curve flattens around the optimum, indicating that higher preservation effort reduces deterioration-related costs and improves system robustness. However, beyond a certain level, further increases in u yield diminishing cost reductions, highlighting a trade-off between preservation investment and ordering decisions.

Figure 5(b) illustrates the behavior of total cost as a function of preservation effort u for different Q . For each fixed Q , the total cost exhibits a well-defined minimum with respect to u , confirming the convexity of the cost function in preservation effort. Increasing u initially reduces total cost due to lower deterioration, but beyond the optimal level the marginal cost of preservation dominates, causing total cost to rise. The location and depth of the minimum vary with Q , highlighting the interaction between ordering and preservation decisions and underscoring the need for joint optimization of u and Q .

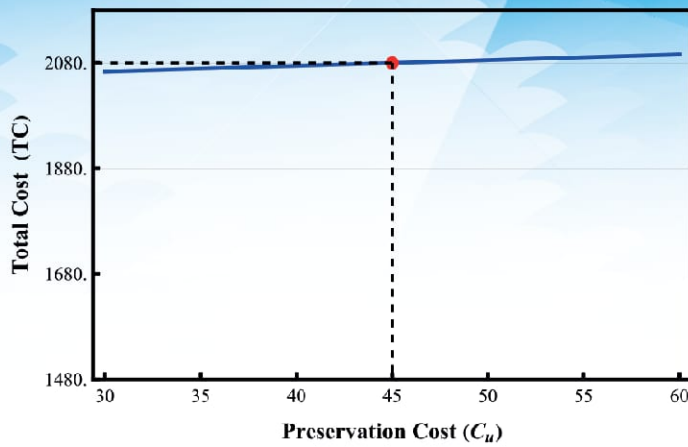


Figure 6(a)

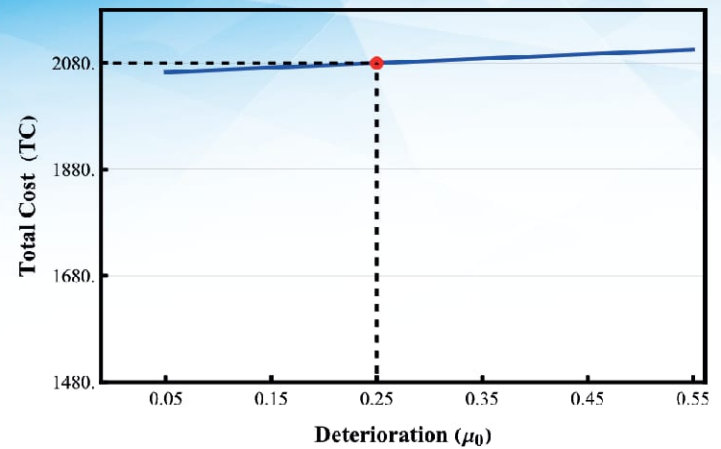


Figure 6(b)

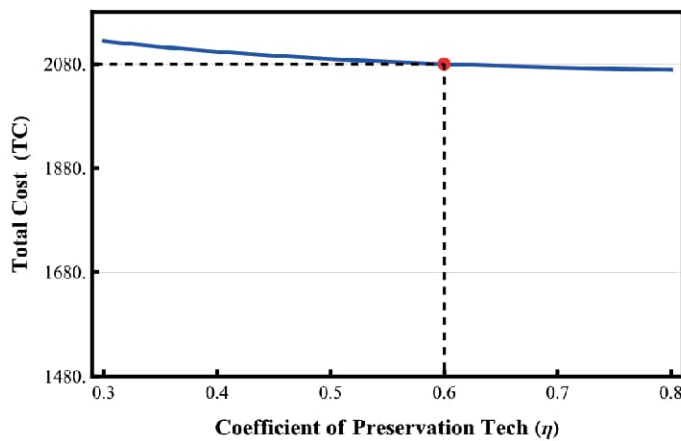


Figure 6(c)

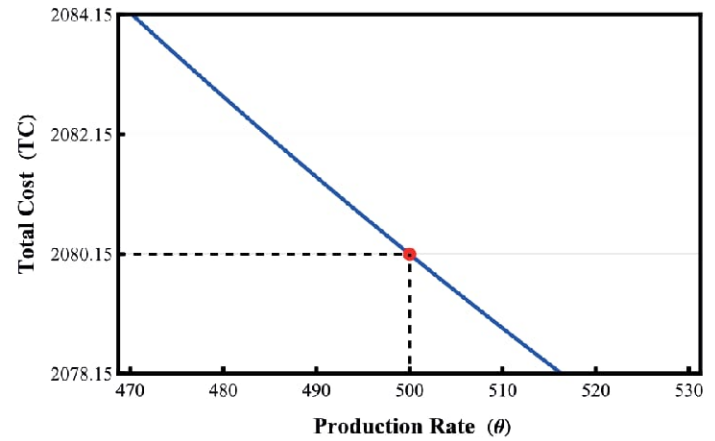


Figure 6(d)

Figure 6(a) represents the impact of the preservation cost parameter C_u on total cost TC. An increase in C_u raises the marginal cost of preservation, which shifts the optimal preservation level downward and leads to a higher minimum total cost. While preservation reduces deterioration-related losses, a higher C_u weakens this trade-off by making each unit of preservation more expensive. Consequently, the cost-minimizing policy relies more on ordering decisions and less on preservation intensity as C_u increases. It is observed from Figure 6(b) that the total cost (TC) is positively sensitive to the baseline deterioration rate μ_0 . An increase in μ_0 intensifies inventory loss and deterioration-related costs, which in turn raises holding and effective production costs. To mitigate this effect, the system responds by increasing the optimal preservation effort and adjusting the order quantity, but these adjustments only partially offset the cost increase. Consequently, higher μ_0 leads to a higher minimum TC, highlighting the economic importance of controlling natural deterioration through technology or operational improvements.

stock accumulation can accelerate deterioration

While Figure 6(c) illustrates that the total cost (TC) is negatively sensitive to the preservation effectiveness parameter η . A higher η enhances the efficiency of preservation investment by accelerating the reduction in the effective deterioration rate for a given preservation effort. This lowers deterioration-related losses and associated holding costs, leading to a reduction in TC up to an economically optimal level. Beyond this range, further increases in η yield diminishing marginal benefits, as preservation gains saturate while investment costs remain. Thus, η plays a critical role in determining the cost-effectiveness of preservation technology and significantly influences the optimal preservation effort and order quantity. Figure 6(d) exhibits a clear sensitivity of total cost function TC to the production rate θ . An increase in θ generally reduces TC by shortening the production and backlog-clearing phases, which lowers shortage-related and

backlog costs and improves system responsiveness to demand. However, beyond a certain level, further increases in θ yield diminishing cost savings, as higher production capacity does not proportionally reduce holding and deterioration-related costs. If production becomes excessively fast relative to demand, it may even increase TC through higher average inventory levels. Hence, θ has a stabilizing but nonlinear impact on TC, and an intermediate production rate is economically optimal rather than an extreme value.

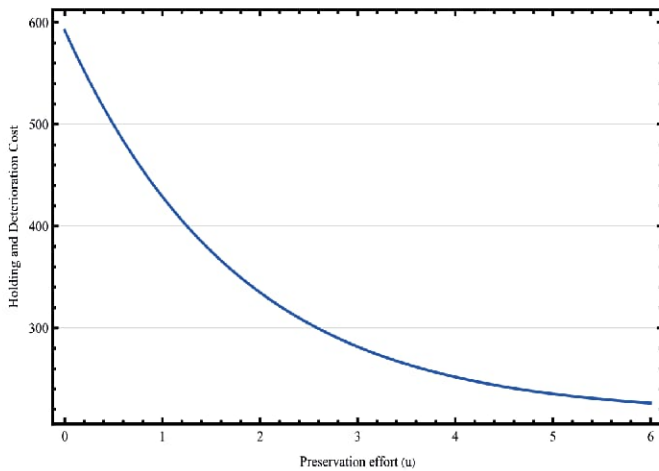


Figure 7: Impact of preservation investment to effective deterioration rate

Figure 7 represents the relationship between preservation effort and the effective deterioration rate. Initial investments in preservation significantly reduce deterioration, but further increases in effort yield progressively smaller improvements, with the deterioration rate approaching a lower bound. This asymptotic behavior indicates that deterioration cannot be fully eliminated through preservation alone. Operationally, this highlights the importance of selecting an optimal preservation level that balances preservation costs against the economic benefits of reduced spoilage, particularly in systems managing perishable or degradable inventories.

6. MANAGERIAL IMPLICATIONS AND PRACTICAL APPLICABILITY

This study provides several actionable insights for firms dealing with deteriorating products under dynamic demand conditions. Firstly, the model highlights that preservation technology is not merely a cost but a strategic lever. By reducing the effective deterioration rate $\mu(u)$, preservation effort

directly extends the usable lifetime of inventory, which in turn: increases the effective sales window, reduces disposal losses and shortens the shortage period. Managers should therefore treat preservation investment as a profit-enabling decision, not a defensive expense. Secondly, the inclusion of recycling (salvage) of deteriorated items introduces a secondary revenue stream. Even when products lose primary market value, partial recovery through recycling at a lower price can significantly offset deterioration losses. This is particularly relevant for industries such as food processing, pharmaceuticals (non-critical use), and electronics refurbishment. The model shows that higher recycling efficiency (ρ) reduces both waste and shortage duration, improving overall system performance. Thirdly, the results emphasize the importance of balancing production and demand under level-dependent consumption. Since demand is influenced by inventory levels, excessive stock accumulation can accelerate deterioration without proportionate revenue gains. Managers should therefore avoid overproduction and instead align production timing with demand responsiveness and preservation capability. Fourthly, the analysis of partial backlogging behavior provides insight into customer tolerance during shortages. The model demonstrates that as waiting time increases, a portion of demand is lost rather than backlogged. This implies that firms must control the shortage interval $T-t_s$ through better preservation and replenishment planning, as prolonged shortages directly translate into lost revenue and reduced customer retention. Fifthly, the integration of recycling cost and disposal cost enables firms to evaluate sustainability trade-offs. When recycling cost is lower than the salvage value, investment in recovery processes becomes economically viable. Conversely, high disposal costs further justify preservation and recycling efforts. Thus, the model supports environmentally conscious decision-making aligned with cost efficiency. Finally, the model is practically applicable to industries dealing with perishable or degradable goods, including: fresh food supply chains, pharmaceuticals and healthcare inventories, chemical and biological products, fashion and seasonal goods, electronic devices with technological obsolescence. In such settings, the joint optimization of production quantity and preservation effort provides a structured framework for minimizing total cost while maintaining service levels and reducing environmental impact.

7. CONCLUSION

This study develops a comprehensive production– inventory model for deteriorating items with level-dependent demand, partial backlogging, and preservation technology. By incorporating preservation investment as a decision variable, the model captures the trade-off between deterioration reduction and additional preservation cost in a realistic and analytically tractable manner. Explicit expressions for the cycle time and total cost are derived, and convexity properties of the total cost function are validated numerically, ensuring the existence and uniqueness of the optimal solution. The numerical experiments and sensitivity analyses demonstrate that preservation effort significantly reduces effective deterioration, though with diminishing returns, and jointly influences the optimal order quantity and total cost. Parameters such as the natural deterioration rate, production rate, and preservation cost are shown to have substantial impacts on system performance, providing clear managerial insights. Overall, the proposed framework offers a robust decision-support tool for managing perishable inventory systems and highlights the economic value of appropriately calibrated preservation strategies under realistic operational conditions.

REFERENCES

- Ghare, P. M. (1963). A model for an exponentially decaying inventory. *J. ind. Engng*, 14, 238-243.
- Karimi, M. (2025). Inventory control of deteriorating items: A review. *International Journal of Industrial Engineering Computations*, 16(1), 93-128.
- Lesmono, D., Limansyah, T., & Sandy, I. A. (2024). An Inventory Model for Deteriorated Item with Time-and Inventory-dependent Demand and Backorder. (IJCSAM) *International Journal of Computing Science and Applied Mathematics*, 10(2), 131-135.
- Goyal, S. K., & Giri, B. C. (2003). The production– inventory problem of a product with time varying demand, production and deterioration rates. *European Journal of Operational Research*, 147(3), 549-557.
- Chang, H. J., & Dye, C. Y. (2001). An inventory model for deteriorating items with partial backlogging and permissible delay in payments. *International Journal of Systems Science*, 32(3), 345-352.
- Sarker, R., Uddin, M. S., Helal, M. A., Khan, A. R., AlArjani, A., & Attia, E. A. (2024). Inventory Model for Instantaneously Deteriorating Items with Multiple Trade Facilities, Stock-and Price-Dependent Demand, and Full Backlogging. *Computation*, 12(12).
- Al Mamun, M. A., & Islam, M. E. (2025). Perishable Production Inventory System with a Level Dependent Demand and offering Discount during Production run. *Journal of Natural Science and Textile Technology*, 4(1), 1.
- Priyamvada, P., Rini, R., & Jaggi, C. K. (2022). Optimal inventory strategies for deteriorating items with price-sensitive investment in preservation technology. *RAIRO-operations Research*, 56(2), 601-617.
- De, P. K., Devi, S. P., & Narang, P. (2024). Inventory model for deteriorating goods with stock and price-dependent demand under inflation and partial backlogging to address post Covid-19 supply chain challenges. *Results in Control and Optimization*, 14, 100369.
- Islam, T., Al Mamun, M. A., & Islam, M. E. (2025). Perishable production inventory model for exponential demand rate with constant production rate and offering discount during production run. *Sonargaon University Journal*, 5(1), 165–180.
- Jagadeeswari, J., & Chenniappan, P. K. (2014). An order level inventory model for deteriorating items with time–quadratic demand and partial backlogging. *Journal of Business and Management Sciences*, 2(3), 79-82.
- Dye, C. Y., Chang, H. J., & Teng, J. T. (2006). A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. *European Journal of Operational Research*, 172(2), 417-429.
- Padmanabhan, G., & VRAT, P. (1990). Inventory model with a mixture of back orders and lost sales. *International Journal of Systems Science*, 21(8), 1721-1726.
- San-José, L. A., Sicilia, J., & García-Laguna, J. (2015). Analysis of an EOQ inventory model with partial backordering and non-linear unit holding cost. *Omega*, 54, 147-157.
- Urban, T. L. (2005). Inventory models with inventory-level-dependent demand: A comprehensive review and unifying theory. *European journal of operational research*, 162(3), 792-804.
- Skouri, K., & Papachristos, S. (2003). Four inventory models for deteriorating items with time varying demand and partial backlogging: A cost comparison. *Optimal Control Applications and Methods*, 24(6), 315-330.
- Balkhi, Z. T., & Benkherouf, L. (2004). On an inventory model for deteriorating items with stock dependent and time-varying demand rates. *Computers & Operations Research*, 31(2), 223-240.

- Mishra, V. K., Singh, L. S., & Kumar, R. (2013). An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. *Journal of industrial engineering international*, 9(1), 4.
- Ouyang, L. Y., Wu, K. S., & Yang, C. T. (2006). A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. *Computers & Industrial Engineering*, 51(4), 637-651.
- Hsu, P. H., Wee, H. M., & Teng, H. M. (2010). Preservation technology investment for deteriorating inventory. *International Journal of Production Economics*, 124(2), 388-394.
- Mishra, U., Cárdenas-Barrón, L. E., Tiwari, S., Shaikh, A. A., & Treviño-Garza, G. (2017). An inventory model under price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. *Annals of operations research*, 254(1), 165-190.
- Zhang, S., Cao, L., & Lu, Z. (2022). An EOQ inventory model for deteriorating items with controllable deterioration rate under stock-dependent demand rate and non-linear holding cost. *Journal of Industrial & Management Optimization*, 18(6).
- Yadav, S., Pareek, S., Sarkar, M., Ma, J. H., & Ahn, Y. H. (2025). A Retail Inventory Model with Promotional Efforts, Preservation Technology Considering Green Technology Investment. *Mathematics*, 13(7), 1065.
- Li, G., He, X., Zhou, J., & Wu, H. (2019). Pricing, replenishment and preservation technology investment decisions for non-instantaneous deteriorating items. *Omega*, 84, 114-126.
- Saha, S., Nielsen, I., & Moon, I. (2017). Optimal retailer investments in green operations and preservation technology for deteriorating items. *Journal of Cleaner Production*, 140, 1514-1527.
- Roy, T., & Chaudhuri, K. S. (2006). Deterministic inventory model for deteriorating items with stock level-dependent demand, shortage, inflation and time discounting. *Nonlinear Phenomena in Complex Systems*, 9(1), 43-52.
- Min, J., Zhou, Y. W., Liu, G. Q., & Wang, S. D. (2012). An EPQ model for deteriorating items with inventory-level-dependent demand and permissible delay in payments. *International Journal of Systems Science*, 43(6), 1039-1053.
- Wu, C., & Zhao, Q. (2014). Supplier-retailer inventory coordination with credit term for inventory $\square$ dependent and linear $\square$ trend demand. *International Transactions in Operational Research*, 21(5), 797-818.
- De, P. K., Devi, S. P., & Narang, P. (2024). Inventory model for deteriorating goods with stock and price-dependent demand under inflation and partial backlogging to address post Covid-19 supply chain challenges. *Results in Control and Optimization*, 14, 100369.
- Kaushik, J. (2024). An inventory model for deteriorating items towards trapezoidal type demand with preservation technology. *Opsearch*, 61(4), 1681-1712.
- Rana, S. K., & Kumar, A. (2024, October). Two-Warehouse Inventory Control for Deteriorating Items Using Hybrid and Stock-Dependent Demand with Partial Backlogging. In *Operations Research Forum* (Vol. 5, No. 4, p. 97). Cham: Springer International Publishing.
- Sharma, S., Tyagi, A., Verma, B. B., & Kumar, S. (2022). An inventory control model for deteriorating items under demand dependent production with time and stock dependent demand. *International Journal of Operations and Quantitative Management*, 27(4), 321-336.