Perishable Production Inventory Model for Exponential Demand Rate with Constant Production Rate and offering Discount during Production Run

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ABSTRACT

This research paper presents a perishable inventory model tailored for items with an exponentially demand rate and constant production rate, incorporating a discount offered during production run. The study assumes an initial buffer stock of inventory, constant replenishment, and zero lead time, with the production rate exceeding demand at any time. As inventory depletes rapidly post-peak level due to demand and deterioration, the model seeks to maximize profit per cycle while accounting for the costs associated with decayed and undecayed inventory. Key factors such as setup cost, holding costs, discount rate, and decay rate are integrated into the model. Through this analysis, we provide insights into optimal production strategies and replenishment for perishable items, contributing to the efficient management of perishable inventories.

Key-words: Perishable inventory, Exponential demand rate, Buffer stock, Inventory level fluctuation, Production run

1.0 INTRODUCTION

In inventory management, maintaining a balance between supply and demand is critical, particularly for perishable goods with finite shelf life. This research focuses on the complexities of managing perishable inventory with an exponential demand, a constant production rate, and the strategic implementation of discounts during production. While traditional inventory models often assume a steady demand rate, real-world scenarios reveal dynamic shifts in demand, especially for perishable products. Our model addresses these complexities by considering a constant production rate, ensuring a sufficient buffer stock at the start of production, and acknowledging the decay rate in inventory. By integrating these factors, this paper provides a framework to assist decision-makers in determining optimal inventory levels, production rates, and pricing strategies for perishable goods with exponential demand decay.

2.0 LITERATURE REVIEW

The field of inventory management has extensively focused on studying models for perishable items. Early models, like Harris's Economic Order Quantity (EOQ), established fundamental principles for minimizing costs in inventory management systems.

(Padiy et al. 2023) created a comprehensive inventory model that considered the perspectives of suppliers, manufacturers, and buyers when it came to perishable items. The factors influencing the quantity of demand and production rate were represented by triangular fuzzy numbers. Three distinct approaches were employed to defuzzify the total profit function. (Kumar,2021) conducted research on a production inventory model for perishable goods that had partial backlogging and a time-dependent demand rate. Due to the shortage, the backlog was partially accumulated. The unmet demand is piled up and it is seen as a result of the time spent waiting. The objective of their research was to maximize the overall profit during a specific period. To demonstrate the practicality of the model, a numerical example was provided. (Manna et al., 2020) explored a model for managing selling price-discount and warranty period dependent demand in a production inventory system, taking into account inspection errors and time-dependent development costs.

In order to investigate the effects of heavy oil exploitation utilizing the PIFB 2018 as a fiscal policy, (Nwosi-Anele et al., 2020) investigated the economics of an onshore heavy oilfield. According to the stochastic results, the quantity of capital investment, the discount rate, and the price of heavy oil had the most effects on the output variables. Furthermore, (Thangam, 2012) explored optimal price discounting and lot-sizing policies for perishable items in a supply chain under specific payment schemes. Mathematical theorems were developed to determine optimal price discounting and lot-sizing policies and a lot of managerial phenomena were obtained through numerical examples.For assessing road infrastructure projects in Germany, (Goldmann, 2019) proposed a social discount rate that accounts for production efficiency, systematic traffic demand risk, and long-term uncertainty.

An integrated inventory policy was presented (Dolgui et al., 2018) for perishable goods in a multi-stage supply chain. The exponential deterioration rate was particularly considered to be in line with the growth rate of the deterioration-causing microorganisms. Using an evolutionary technique such as a genetic algorithm, they demonstrated how to optimize an integrated model that incorporates inventory control and fleet selection. And there are other things to think about in addition to that. In addition, (Tekin and Erol, 2017) indicated that perishable goods have a limited lifespan, as their shelf lives are relatively short. Product efficiency decreases when non-recyclable items are discarded without being utilized. Additionally, these types of products that occupy the shelves of supermarkets lead supermarkets to adopt an inadequate stock management policy.

(Islam, 2016) created a time-dependent inventory model using a constant production rate and market demands that were decreasing exponentially. The model also took into account the minimal decay. The goal of their research was to determine the ideal inventory cost and the most efficient time cycle. The model was also justified by demonstrating its convexity and providing a numerical example. (Cao and Xie, 2015) analyzed a stochastic inventory system where the manufacturer aimed to maximize the long-term average profit by adjusting the selling price and controlling the production process.

Furthermore, (Schlosser, 2015) analyzed a specific category of dynamic pricing and advertising models, focusing on the sale of perishable goods. He considered factors such as marginal unit costs and inventory holding costs. They assumed that the time period was limited and allowed certain model parameters to vary based on time.

With the goal of identifying the ideal replenishment cycle, (Tayal et al., 2015) created an EPQ model for non-instantaneous deteriorating commodities with time-dependent holding costs and exponential demand rates. (Pattnaik, 2011) developed a non-linear optimal inventory policy for perishable goods that deteriorate quickly and offer price discounts, demonstrating how the discounted approach yields the best outcomes. (Panda, 2009) conducted a study on a single item economic order quantity model, where the demand is influenced by the stock level. It was commonly believed that after a specific period, the product begins to degrade and due to visual cues and other factors of deterioration, the demand remains constant. On that document, they examined the potential discount on the selling price that could be offered during deterioration to maximize the profit per unit of time and whether a pre-deterioration discount influences the profit per unit or not.

(Maity and Maiti, 2008) examined an inventory management system for deteriorating multi-items that were under the supervision of a single manager. In an environment with limited resources and the effects of inflation, the decision-making process became more challenging.

(Oganezov, Karen, 2006) suggested an inventory system for economic production models in paper in his research work. The goal was to determine the ideal cycle time that minimizes the overall cost and, if permitted, the ideal degree of shortfall. When creating various models, a number of factors are taken into account, including the time value of money, inflation, constant and linear demand rates, shortages, and deterioration.

In order to describe a general model in which the production rate, the product demand rate, and the item deterioration rate are all taken into consideration as functions of time, (Yan, H., & Cheng, 1998) looked into a perishable single item production-inventory system. They also discussed the best times to stop and restart production in order to minimize the total relevant cost per unit time. They believed that a demand deficit, in which part of the demand is lost and the remainder is backlogged, is acceptable. As instructive examples, the most basic scenarios with constant rates of production, demand, and deterioration were examined.

In an inventory model put forth by (Sarker et al., 1997) demand was viewed as a composite function made up of a variable component that is proportionate to the inventory level during times of positive inventory buildup and a constant component. Additionally, it was believed that the rate of decay was exponential and the rate of production was finite.

3.0 ASSUMPTIONS

- Production begins with a fixed quantity of inventory as buffer stock.
- Production rate is constant at any time.
- The demand rate falls exponentially.
- There is no lead time.
- Inventory levels peak at a certain time and then rapidly decline as a result of degradation and demand.
- Shortage are not permitted
- The buffer stock can be replenished indefinitely at the end of the cycle

3.1 Notations

- γ : Production rate which is constant and greater than demand rate at any time.
- $\beta e^{-\varepsilon t}$: Demand rate at any instant t .
- β: Initial demand where $\beta = 0, 1, 2, 3, \ldots \ldots$
- ε: Increment of demand where $0 < \varepsilon < 1$
- ω: Decay rate for unit inventory which is very small and constant.
- Q: Inventory level at time $t = 0$ and represents the buffer stock.
- Q_1: Inventory level at time $t = t_1$.
- dt: very small portin of instant t.
- k: Set up cost.
- h_1:Holding cost of un-decayed inventory at time $t = 0$ to T.
- h_2:Holding cost of deteriorating inventory at time $t = 0$ to T.
- α: Discount rate applicable during production.

- t_1:Time for reaching inventory at maximum level.
- T: Length of total cycle.
- $I_1(t)$: Undecayed inventory at time $= 0\ to\ t_1$.
- $I_2(t)$:Undecayed inventory at time $=\ t_1 to\ T.$.
- τ_1 :Deteriorated inventory at $t = 0$ to t_1.
- τ_2: Deteriorated inventory at $t = t_1$ to T.
- $TP(Q)$: Profit Function per cycle.

3.2 Mathematical Model

The model is developed on the basis of exponential market demands and constant production capacity of the organization. The model is suitable for the products which have finite shelf-life and ultimately causes the products decay. At the beginning, i.e. at time t = 0 , the production starts with Q inventory. In this model, the production rate γ remains uniform for entire production cycle. The proposed inventory is shown below in figure 01.

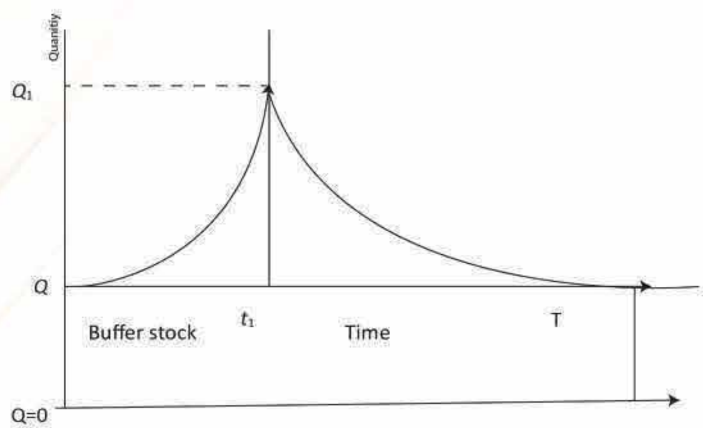

Figure 01: Proposed Inventory model under study

But the demands exponentially decrease time to time, which is shown in the Figure 01. During time $t = o\ to\ T$.The inventory decreases at the rate of $\gamma - \beta e^{-\epsilon t} - \omega I(t), as\ \beta e^{-\epsilon t}$ is the market demand. Here, $\omega I(t)$ is the decay of *I (t)* inventories at instant t . By using the above arguments we can have the following differential equations,

$$I(t + dt) = I(t) + \{\gamma - \beta e^{-\varepsilon t}\}dt - \omega I(t)dt$$

$$\lim_{dt\to 0} \frac{I(t+dt)-I(t)}{dt} = \{\gamma - \beta e^{-\varepsilon t} - \omega I(t)\}$$

$$\frac{d}{dt}I(t) + \omega I(t) = \gamma - \beta e^{-\varepsilon t}$$

The general solution of the differential equation is $I(t) = \frac{\gamma}{\omega} - \frac{\beta}{\omega-\epsilon}e^{-\varepsilon t} + Ae^{-\omega t}$

When the boundary condition is applied, that is, when $t = 0, I(t) = Q$, we obtain,

$$A = Q - \frac{\gamma}{\omega} + \frac{\beta}{\omega-\varepsilon}$$

Consequently, $I(t) = \frac{\gamma}{\omega} - \frac{\beta}{\omega-\varepsilon}e^{-\varepsilon t} + \left(Q - \frac{\gamma}{\omega} + \frac{\beta}{\omega-\varepsilon}\right)e^{-\omega t}$

$$= \frac{\gamma}{\omega}(1 - e^{-\omega t}) + \frac{\beta}{\omega-\varepsilon}(e^{-\omega t} - e^{-\varepsilon t}) + Qe^{-\omega t} \qquad (1)$$

When we take up to the first degree of ω from the other boundary condition, that is, at $t = t_1, I(t) = Q_1$,we get following equation:

$$Q_1 = \frac{\gamma}{\omega}(1 - e^{-\omega t_1}) + \frac{\beta}{\omega-\epsilon}(e^{-\omega t_1} - e^{-\varepsilon t_1}) + Qe^{-\omega t_1}$$

$$= \frac{\gamma}{\omega}(1 - 1 + \omega t_1) + \frac{\beta}{\omega-\varepsilon}(-\omega t_1 + \varepsilon t_1) + Q(1 - \omega t_1)$$

$$= \gamma t_1 - \beta t_1 + Q - \omega Q t_1 \qquad (2)$$

For convenience, the undecayed inventory during $t = 0$ to t_1, is calculated using equation (1) and taking into account up to the second degree of ω.

$$I_1 = \int_0^{t_1} I(t)dt$$

$$= \int_0^{t_1}\{\frac{\gamma}{\omega}(1 - e^{-\omega t}) + \frac{\beta}{\omega-\epsilon}(e^{-\omega t} - e^{-\varepsilon t}) + Qe^{-\omega t}\}dt$$

$$= \frac{\gamma}{\omega}[t + \frac{1}{\omega}(e^{-\omega t})]_0^{t_1} + \frac{\beta}{\omega-\varepsilon}[\frac{e^{-\omega t}}{-\omega} + \frac{e^{-\varepsilon t}}{\varepsilon}]_0^{t_1} + Q[\frac{e^{-\omega t}}{-\omega}]_0^{t_1}$$

$$= \frac{\gamma}{\omega}\left\{t_1 + \frac{1}{\omega}(e^{-\omega t_1} - 1)\right\} + \frac{\beta}{\omega-\varepsilon}\left\{\frac{e^{-\omega t_1-1}}{-\omega} + \frac{e^{-\varepsilon t_1-1}}{\varepsilon}\right\} - \frac{Q}{\omega}(e^{-\omega t_1} - 1)$$

$$=\frac{\gamma}{\omega}\left\{t_1+\frac{1}{\omega}\left(1-\omega t_1+\frac{\omega^2 t_1{}^2}{2}-1\right)\right\}+\frac{\beta}{\omega-\varepsilon}\left\{\frac{1}{-\omega}\left(-\omega t_1+\frac{\omega^2 t_1{}^2}{2}\right)+\frac{1}{\varepsilon}\left(-\varepsilon t_1+\frac{\varepsilon^2 t_1{}^2}{2}\right)\right\}-\frac{Q}{\omega}\left(-\omega t_1+\frac{\omega^2 t_1{}^2}{2}\right)$$

$$=\frac{\gamma t_1{}^2}{2}+\frac{\beta}{\omega-\varepsilon}\left(t_1+\frac{\varepsilon t_1{}^2}{2}-t_1-\frac{\omega t_1{}^2}{2}\right)+Qt_1-\frac{Q\omega t_1{}^2}{2}$$

$$=Qt_1+\frac{\gamma t_1{}^2}{2}-\frac{\beta t_1{}^2}{2}-\frac{Q\omega t_1{}^2}{2}$$

$$=Qt_1+\frac{1}{2}(\gamma-\beta-Q\omega)t_1{}^2$$

$$\therefore I_1=\ Qt_1+\frac{1}{2}(\gamma-\beta-Q\omega)t_1{}^2 \qquad (3)$$

Now the deteriorating items during the period considering the decay of items is as follows:

$$\tau_1=\int_0^{t_1}\omega\, I(t)dt$$

$$=\omega\, Qt_1+\frac{1}{2}\omega(\gamma-\beta-Q\omega)t_1{}^2 \qquad (4)$$

However, since there is no production after time t_1, the inventory falls at the rate of

$-\beta e^{-\varepsilon t}-\omega I(t)$ during $t=t_1$ to T_1 .The market's demand and the items' deterioration cause the inventory to run out.

It is now possible to create another differential equation in the same manner as previously:

$$\frac{d}{dt}I(t)+\omega I(t)=-\beta e^{-\varepsilon t}$$

The following defines general solution of the differential equation :

$$I(t)=-\frac{\beta}{\omega-\epsilon}e^{-\varepsilon t}+Be^{-\omega t}$$

Applying the boundary condtion, i.e. at $t=T,\ I(t)=Q$

By solving we get $B=Qe^{\omega t}+\frac{\beta}{\omega-\varepsilon}e^{(\omega-\varepsilon)T}$

Therefore , $I(t)=-\frac{\beta}{\omega-\varepsilon}e^{-\varepsilon t}+Qe^{(T-t)\omega}+\frac{\beta}{\omega-\varepsilon}e^{(\omega-\varepsilon)T-\omega t}$ (5)

Taking up to the first order of ω and substituting another boundary condition, that is

at $t=t_1, I(t)=Q_1$,we obtain the following equation,

$$Q_1=-\frac{\beta}{\omega-\varepsilon}e^{-\varepsilon t_1}+Qe^{(T-t_1)\omega}+\frac{\beta}{\omega-\varepsilon}e^{(\omega-\varepsilon)T-\omega t_1}$$

$$=\frac{\beta}{\omega-\varepsilon}\left\{e^{(\omega-\varepsilon)T-\omega t_1}-e^{-\varepsilon t_1}\right\}+Qe^{(T-t_1)\omega}$$

$$= \frac{\beta}{\omega-\varepsilon}(\omega T - \varepsilon T - \omega t_1 + \varepsilon t_1) + Q(1 + T\omega - \omega t_1)$$

$$= \beta(T - t_1) + Q(1 + T\omega - \omega t_1)$$

$$\therefore Q_1 = \beta(T - t_1) + Q(1 + T\omega - \omega t_1) \qquad (6)$$

Now, we obtain the un-deayed inventory during $t = t_1$ to T_1as follows using equation (5) and taking into account up to the first degree of φ:

$$I_2 = \int_{t_1}^{T} I(t)dt$$

$$= \int_{t_1}^{T} \{-\frac{\beta}{\omega-\varepsilon} e^{-\varepsilon t} + Qe^{(T-t)\omega} + \frac{\beta}{\omega-\varepsilon} e^{(\omega-\varepsilon)T-\omega t}\}dt$$

$$= \frac{\beta}{\varepsilon(\omega-\varepsilon)}[e^{-\varepsilon t}]_{t_1}^{T} - \frac{Q}{\omega}[e^{(T-t)\omega}]_{t_1}^{T} - \frac{\beta}{\omega(\omega-\varepsilon)}[e^{(\omega-\varepsilon)T-\omega t}]_{t_1}^{T}$$

$$= Q(T - t_1)$$

$$\therefore I_2 = Q(T - t_1) \qquad (7)$$

Considering the decay of the items, we calculate the deteriorating items as below:

$$\delta_2 = \int_{t_1}^{T} \omega I(t)dt$$

$$\delta_2 = Q\omega(T - t_1) \qquad (8)$$

We get from equation (2) and (6)

$$\gamma t_1 - \beta t_1 + Q - \omega Q t_1 = \beta(T - t_1) + Q(1 + T\omega - \omega t_1)$$

$$T = \frac{\gamma t_1}{\beta + \omega Q} \qquad (9)$$

Total Cost Function

The cost function can be described in the following form:

$$TC = \frac{1}{T}\{k + h_1(I_1 + I_2) + h_2(\tau_1 + \tau_2) + h_3\delta_3 + \alpha p I_1\}$$

by using equation (3), (4), (7) and (8)

$$TC = \frac{1}{T}\left[\alpha p Q t_1 + \frac{1}{2}\{(h_1 + \alpha p + h_2\omega)(\gamma - \beta - \omega Q\}t_1^2 + \mathrm{k}\right] + Q(h_1 + h_2\omega) \qquad (10)$$

Total Revenue Function

Total revenue function is as follows:

$$TR = \frac{1}{T}[P(I_1 + I_2)]$$

Using equation (3) and (7)

$$TR = \frac{p}{2T}(\gamma - \beta - \omega Q)t_1^2 + pQ \tag{11}$$

Total Profit Function

Total profit function can be obtained by subtracting equation (11) from (10)

$$TP = TR - TC$$

$$= \frac{1}{T}\left[-\alpha pQt_1 + \frac{1}{2}\{(p - h_1 - \alpha p - h_2\omega)(\gamma - \beta - \omega Q)\}t_1^2\right] - \frac{k}{T} + Q(p - h_1 - h_2\omega)$$

By using equation (9)

$$TP(Q,t_1) = \frac{-\alpha pQ(\beta+\omega Q)}{\gamma} + \frac{(\beta+\omega Q)}{2\gamma}(p - h_1 - \alpha p - h_2\omega)(\gamma - \beta - \omega Q)t_1 - \frac{k(\beta+\omega Q)}{\gamma t_1} + Q(p - h_1 - h_2\omega)$$

(12)

Now differentiating TP partially with respect to t_1 and equate to zero we get

$$\frac{\partial TP}{\partial t_1} = 0$$

$$\frac{-(p - h_1 - \alpha p - h_2\omega)(\beta - \gamma + \omega Q)}{2} + \frac{k}{{t_1}^2} = 0$$

$$\therefore \mathrm{t}_1 = \frac{\sqrt{2\mathrm{k}}}{\sqrt{(p-h_1-\alpha p-h_2\omega)(\beta-\gamma+\omega Q)}} \tag{12}$$

We get from equation (12)

$$TP(Q) = \frac{-1}{\gamma}\left\{\alpha p\beta Q + \alpha p\omega Q^2 + \frac{(\beta + \omega Q)}{\gamma}\sqrt{2k(p - h_1 - \alpha p - h_2\omega)(\beta - \gamma + \omega Q)}\right\} + Q(p - h_1 - h_2\omega) \tag{13}$$

By differentiating (13) with respect to Q and equate to zero, we get

$$8(\alpha p)^2\omega^3Q^3 + \omega^2[8(\alpha p)^2(\beta - \gamma) - 8\alpha p\{\gamma(p - h_1 - h_2\omega) - \alpha\beta p\}$$
$$-9k(p - \alpha p - h_1 - h_2\omega)\omega^2]Q^2 + \omega[2\{\gamma(p - h_1 - h_2\omega) - \alpha\beta p\}^2 - 8\alpha p(\beta - \gamma)$$
$$\{\gamma(p - h_1 - h_2\omega) - \alpha\beta p\} - 6k(p - \alpha p - h_1 - h_2\omega)\omega^2(3p - 2\gamma)]Q + 2(\beta - \gamma)$$
$$\{\gamma(p - h_1 - h_2\omega) - \alpha\beta p\}^2 - k(p - \alpha p - h_1 - h_2\omega)\{\omega(3\beta - 2\gamma)\}^2 = 0$$

We can write the above equation in the following form

$$aQ^3 + bQ^2 + cQ + d = 0 \tag{14}$$

where $a = 8(\alpha p)^2\omega^3$

$$b = \omega^2[8(\alpha p)^2(\beta-\gamma) - 8\alpha p\{\gamma(p-h_1-h_2\omega)-\alpha\beta p\} - 9k(p-\alpha p-h_1-h_2\omega)\omega^2$$

$$c = \omega[2\{\gamma(p-h_1-h_2\omega)-\alpha\beta p\}^2 - 8\alpha(\beta-\gamma)\{\gamma(p-h_1-h_2\omega)-\alpha\beta p\}$$

$$-6k(p-\alpha p-h_1-h_2\omega)\omega^2(3p-2\gamma)$$

$$d = 2(\beta-\gamma)\{\gamma(p-h_1-h_2\omega)-\alpha\beta p\}^2 - k(p-\alpha p-h_1-h_2\omega)\{\omega(3\beta-2\gamma)\}^2$$

Solving equation (14), we obtain Q as:

$$Q = \sqrt[3]{-\frac{s}{2}+\sqrt{(\frac{s}{2})^2+(\frac{r}{3})^3}} + \sqrt[3]{-\frac{s}{2}-\sqrt{(\frac{s}{2})^2+(\frac{r}{3})^3}} - \frac{b}{3a} \tag{15}$$

Where $r = \frac{3ac-b^2}{3a^2}$ and $s = \frac{2b^3-9abc+27a^2d}{27a^3}$

4.0 NUMERICAL ILLUSTRATION

In this section we derived numerical illustrations by taking the values of the parameters involve in this model as follows:

$\gamma = 3, \beta = 1, \alpha = 0.1, \epsilon = 0.02, \omega = 0.01, h_1 = 1, h_2 = 1.2, p = 10, k = 100$. By using the equation (9), (12) ,(13) and (15) , then we get the results as optimum order interval

$T^* = 1.58696$, optimum time $t_1{}^* = 0.367871$, maximum order quantity

$Q^* = 1994.17$ and total average optimum profit $TP^* = 5038.0012$

Details are shown in the Table 01 and Figure 02

Table 01. Impact of Quantity , t_1 and T

Q^*	${t_1}^*$	T^*	TP^*
1191.17	0.366468	1.58457	5037.9698
1192.17	0.366934	1.58536	5037.9873
1193.17	0.367402	1.58616	5038.0011
1194.17	0.367871	1.58696	5038.0012
1195.17	0.368341	1.58776	5038.0011
1196.17	0.368812	1.58856	5037.9873
1197.17	0.369283	1.58936	5037.9698

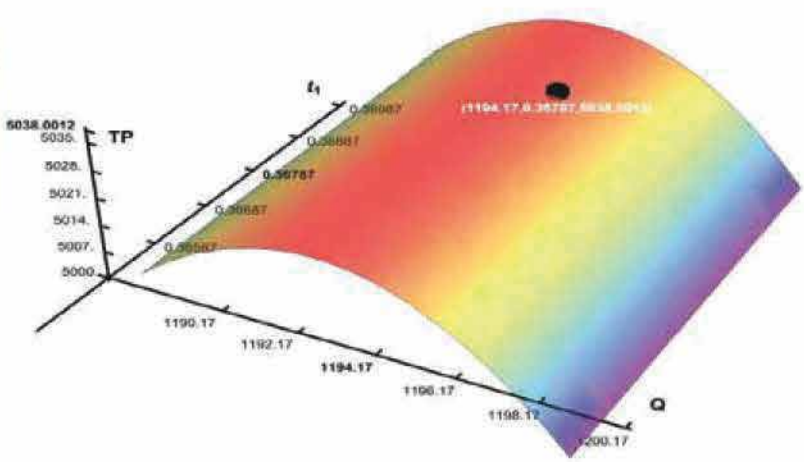

Figure 02: TP and t_1 against Q

From Table 01 and Figure 02, it is observed that as Q increases, the total profit generally rises because higher production levels generate more revenue. This is particularly true when the selling price and other parameters like demand are relatively high. Beyond a certain point, the increase in Q may lead to falling returns on profit. This happens because of increasing costs, such as holding costs, decay rates, and other variable costs that escalate with higher production levels. As these costs grow, they begin to offset the additional revenue gained from producing more.

5.0 SENSITIVITY ANALYSIS

Now, in this section we are going to illustrate how the inventory system or the solution is affected by even a little changes of parameters $\gamma, \beta, \alpha, \epsilon, \omega, h_1, h_2, p, k$

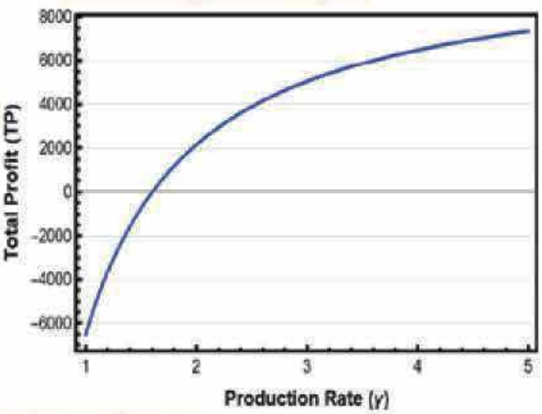

Figure 03(a): Production Rate verses Total Profit

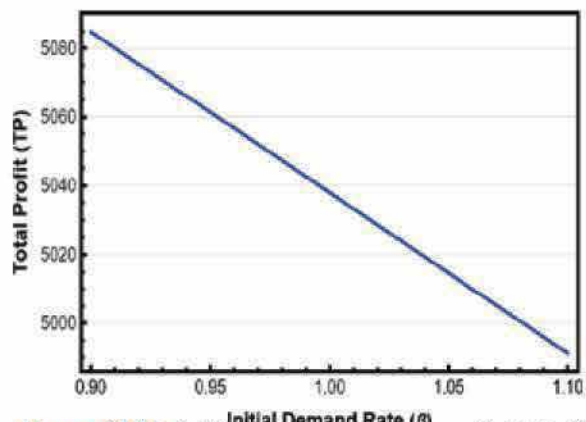

Figure 03(b): Initial Demand Rate verses Total Profit

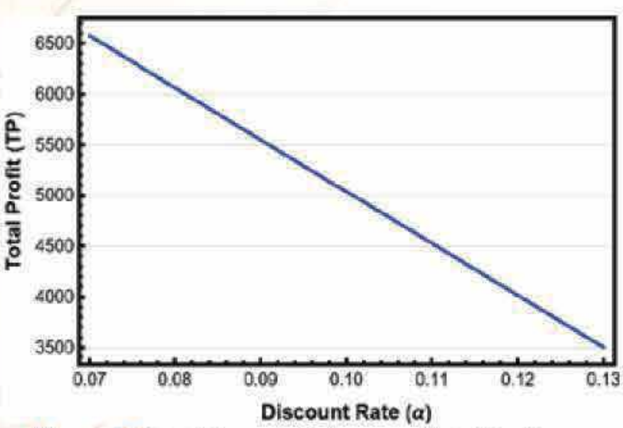

Figure 03(c): Discount Rate verses Total Profit

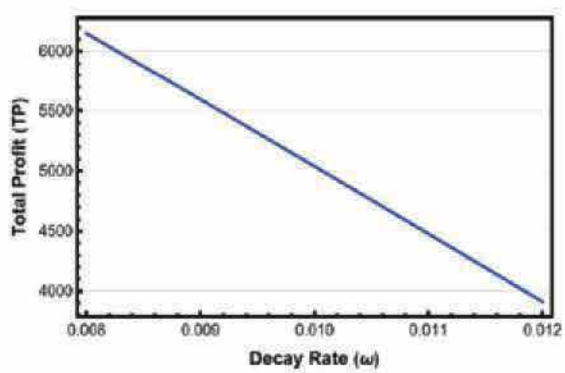

Figure 03(d): Decay Rate verses Total Profit

Figure 03(a) informs us about the impact of production rate γ on total profit. When γ is sufficiently high, then production meets or exceed the demand rate, ensures stability in supply. This helps achieve higher profit margins.On the other hand extremely high values of γ might lead to increased production costs or resource usage, potentially diminishing overall profitability. Thus, an optimal range for γ exists where profit is maximized without incurring excessive holding or production costs. Therefore a balanced and adequately high production rate maximizes profit by sustaining inventory levels and meeting demand effectively while controlling holding and deterioration costs.

It is observed from figure 03(b) that a higher β value signifies a stronger initial demand, which can lead to increased sales and thus, a potential increase in profit.With an increased β, the production rate may need to be adjusted to maintain sufficient inventory levels to satisfy the higher demand. This may lead to higher production and holding costs, especially if demand decreases over time (due to the exponential decay factor). Balancing with production costs is crucial to optimize profit.

Figure 03(c) illustrates that as α increases, the discounted value of revenues generated from each unit sold during production, which lowers the total profit. This negative relationship is often reflected by a downward trend in the $TP(Q)$ graph as α rises. The graph shows a steep decline suggesting that total profit is highly sensitive to changes in the discount rate. Even a small increase in α could lead to a considerable drop in profit, making it essential to manage α carefully.

It is found in the figure 03(d) that increase in ω, the cost of holding deteriorating inventory (represented by h_2) also rises. This leads to a higher overall cost structure since a larger portion of the inventory incurs these additional holding costs, impacting profit margins.Thus profit function $TP(Q)$ is highly sensitive to changes in ω. A small increase in the decay rate can lead to a disproportionately large decrease in profit, especially if the demand rate does not decrease at a similar rate. Balancing with demand and replenishment strategies is essential to avoid high decay costs while meeting customer needs.Keeping ω low helps in maximizing $TP(Q)$ by preserving more of the inventory for sale rather than decay.

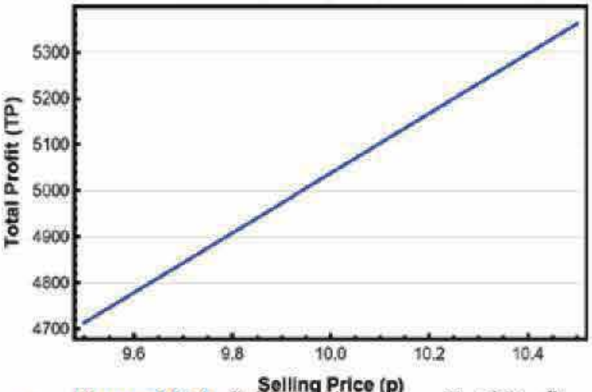

Figure 04(a): Selling Price verses Total Profit

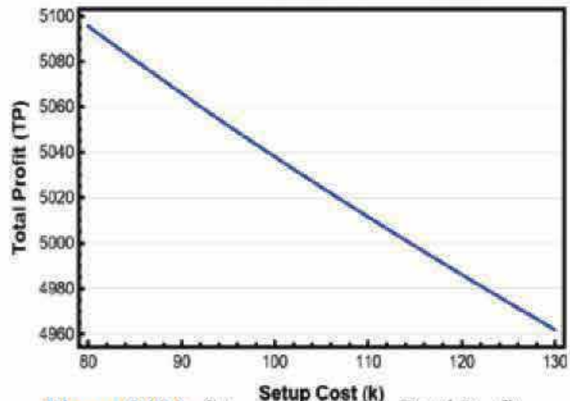

Figure 04(b): Set up cost verses Total Profit

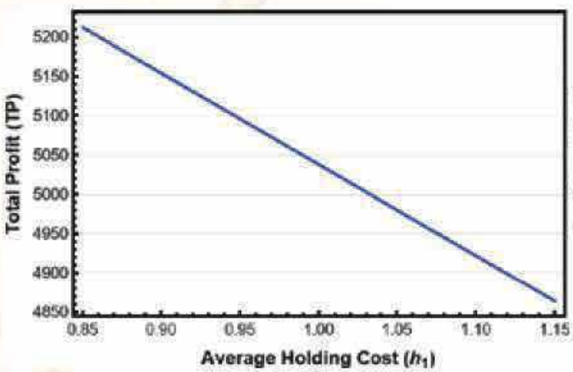

Figure 04(c): Holding cost (h_1) verses Total Profit

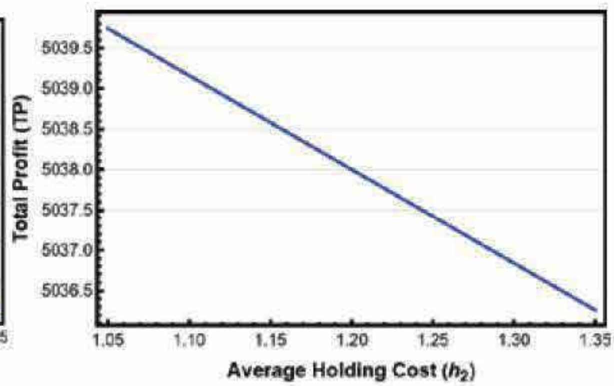

Figure 04(d): Holding cost (h_2) verses Total Profit

figure 04(a) visualized the impact of selling price on profit. The graph showed an upward trend, indicating that higher selling prices boost the total profit. On the contrary, at very high values of p, the rate of increase in $TP(Q)$ might begin to slow due to factors such as market demand elasticity, where demand decreases as prices rise, or cost constraints, which offset the revenue benefits of higher prices.

It is noticed from figure 04(b) that, as setup cost k increases, the total profit $TP(Q)$ decreases. The graph often shows a downward trend, reflecting that higher setup costs reduce the profit margin by increasing overall costs per cycle. So reducing setup costs can significantly boost profitability. Businesses can maximize $TP(Q)$ by maintaining k within an optimal range, ensuring production efficiency and effective cost management to sustain profitability over time.

The downward trend of figure 04(c) indicating that higher holding costs reduce profit per cycle. This aligns with the idea that higher storage costs eat into profit margins. Therefore by managing h_1 within an optimal range, businesses can better control inventory costs, maximizing $TP(Q)$ and ensuring cost-effective inventory practices.

We observed from figure 04(d) that an increase in h_2 generally results in a decrease in $TP(Q)$. As h_2 rises, the profit per cycle is reduced, since the holding costs for deteriorating inventory represent an expense that directly reduces net profit. Keeping h_2 within a manageable range helps in sustaining profitability. Businesses should prioritize inventory management techniques that reduce the amount and duration of deteriorating stock in inventory, thus lowering h_2 and maximizing $TP(Q)$.

6.0 CONCLUSION

This paper presents an advanced inventory model that combines exponential demand decay, constant production, and discount strategies to enhance profit in perishable inventory systems. By incorporating factors such as decay rate, discount rate, setup cost, and holding costs for decayed and undecayed inventory, the model offers a practical solution for optimizing inventory management in the perishable goods industry. The results demonstrate that inventory systems can be better managed through a well-defined replenishment policy, effectively balancing production rates and demand decline. The model also highlights the value of discounting during production to maximize profitability in scenarios with perishable goods. Future research could explore dynamic adjustments to the discount rate and production rate to further enhance the model's applicability across various industries.

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